

A Kilonova Toy Model

How to cook a simple Kilonova

Chef: Stefano Ascenzi



Model based on *Metzger 2020, LRR,*
23, 1, Section 4.



1

Ingredients

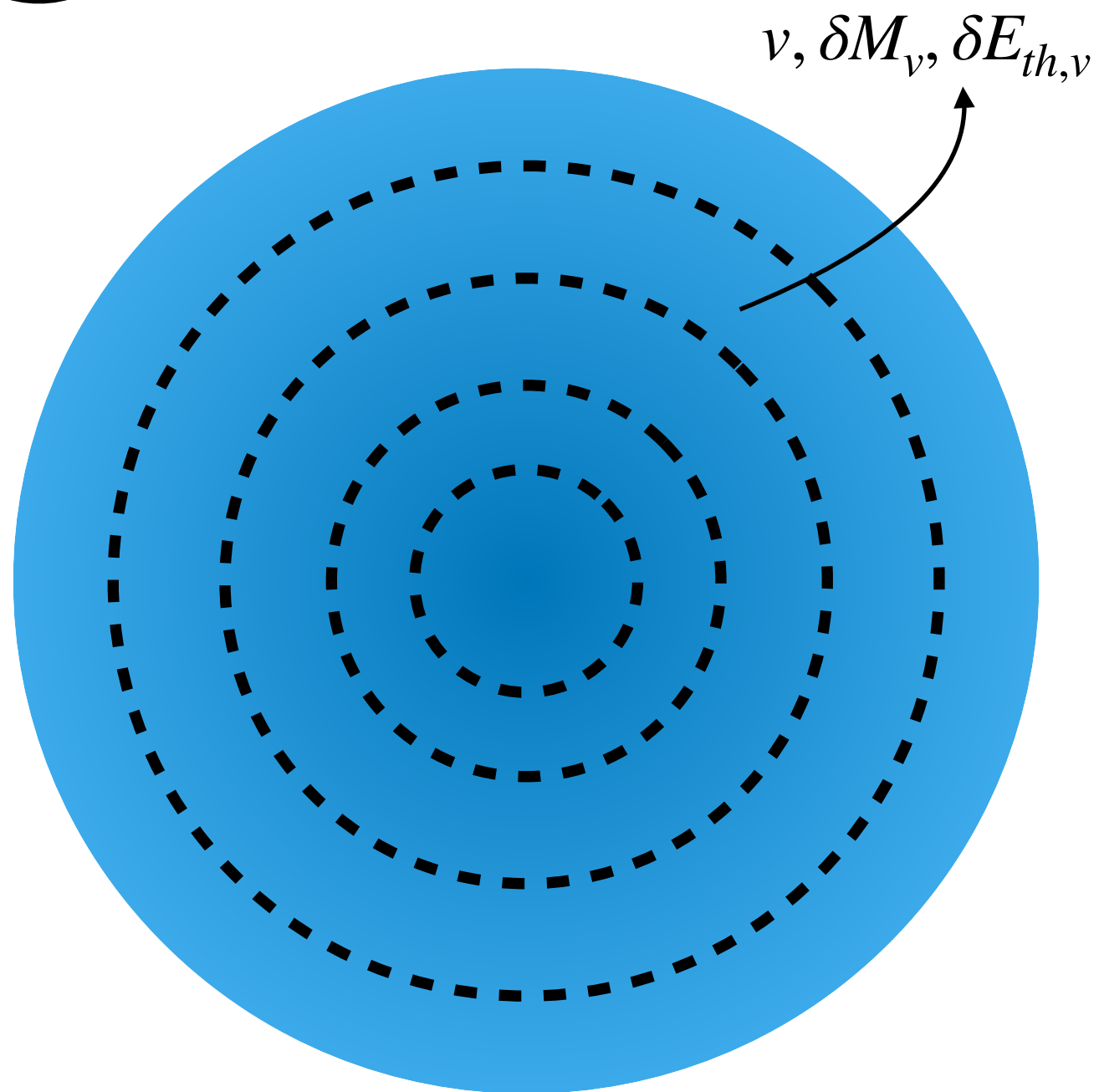
We start with a given mass M of material with electron fraction Y_e expanding with velocities in the interval $[v_0, v_{max}]$

The matter is in homologous expansion, namely: $v \propto R$

The mass is distributed as a power-law of velocity, such that the mass with a velocity higher than v is:

$$M_v = M \left(\frac{v}{v_0} \right)^{-\delta}$$

2



The thermal energy of each shell evolves according to the energy conservation:

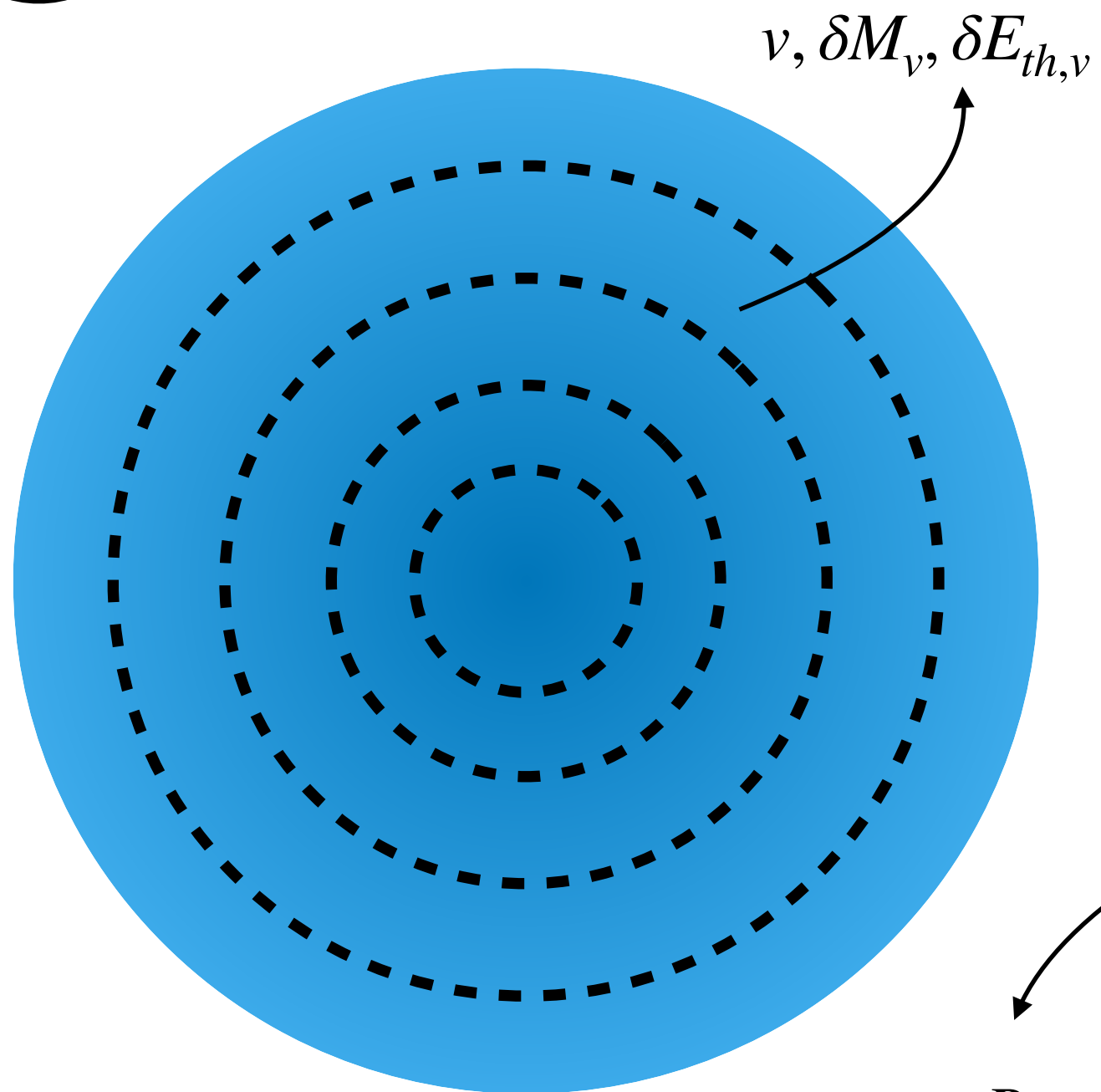
$$\frac{d}{dt}(\delta E_{th,\nu}) = - \frac{\delta E_{th,\nu}}{R_\nu} \frac{dR_\nu}{dt} - L_\nu + Q_\nu$$

Adiabatic loss
($p dV$)

Radiative loss
(what we observe!)

Heating

3



The luminosity in the shell is the thermal energy over the characteristic time (diffusion time/light crossing time)

$$L_\nu = \frac{\delta E_{th,\nu}}{t_{diff,\nu} + t_{lc}}$$

$$t_{diff,\nu} = \tau_\nu \frac{R_\nu}{c} = \frac{3M_\nu k_\nu}{4\pi\beta R_\nu c}$$

$$t_{lc} = \frac{R_\nu}{c}$$

4

We need to solve the equations:

$\nu, \delta M_\nu, \delta E_{th,\nu}$

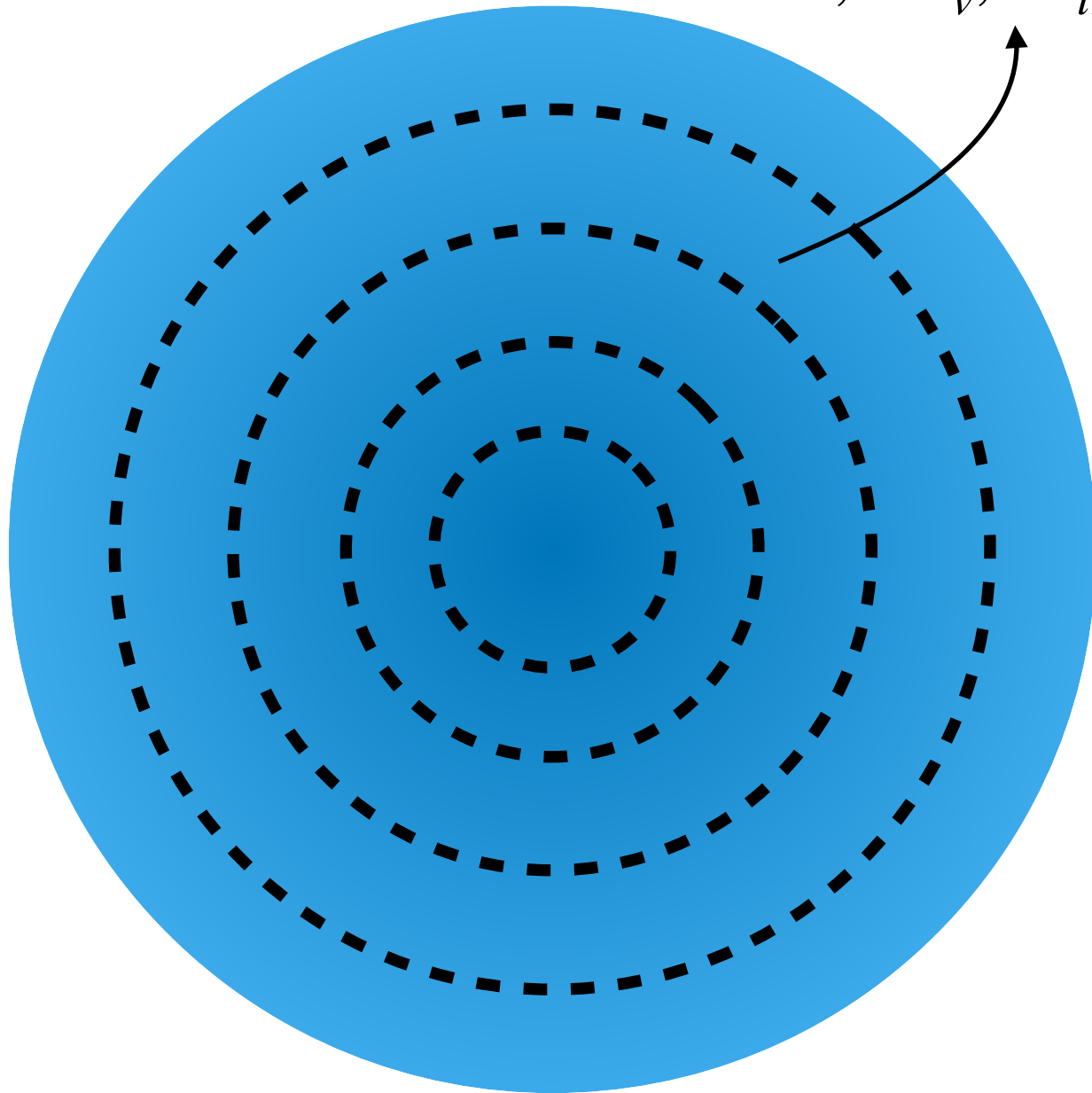
$$\begin{cases} \frac{dR_\nu}{dt} = \nu \\ \frac{d}{dt}(\delta E_{th,\nu}) = -\frac{\delta E_{th,\nu}}{R_\nu} \frac{dR_\nu}{dt} - L_\nu + Q_\nu \end{cases}$$

For $\delta E_{th,\nu}$ to find L_ν

$$L_\nu = \frac{\delta E_{th,\nu}}{t_{diff,\nu} + t_{lc}}$$

And sum over the shell to find the total luminosity

$$L_{tot} = \sum L_\nu$$



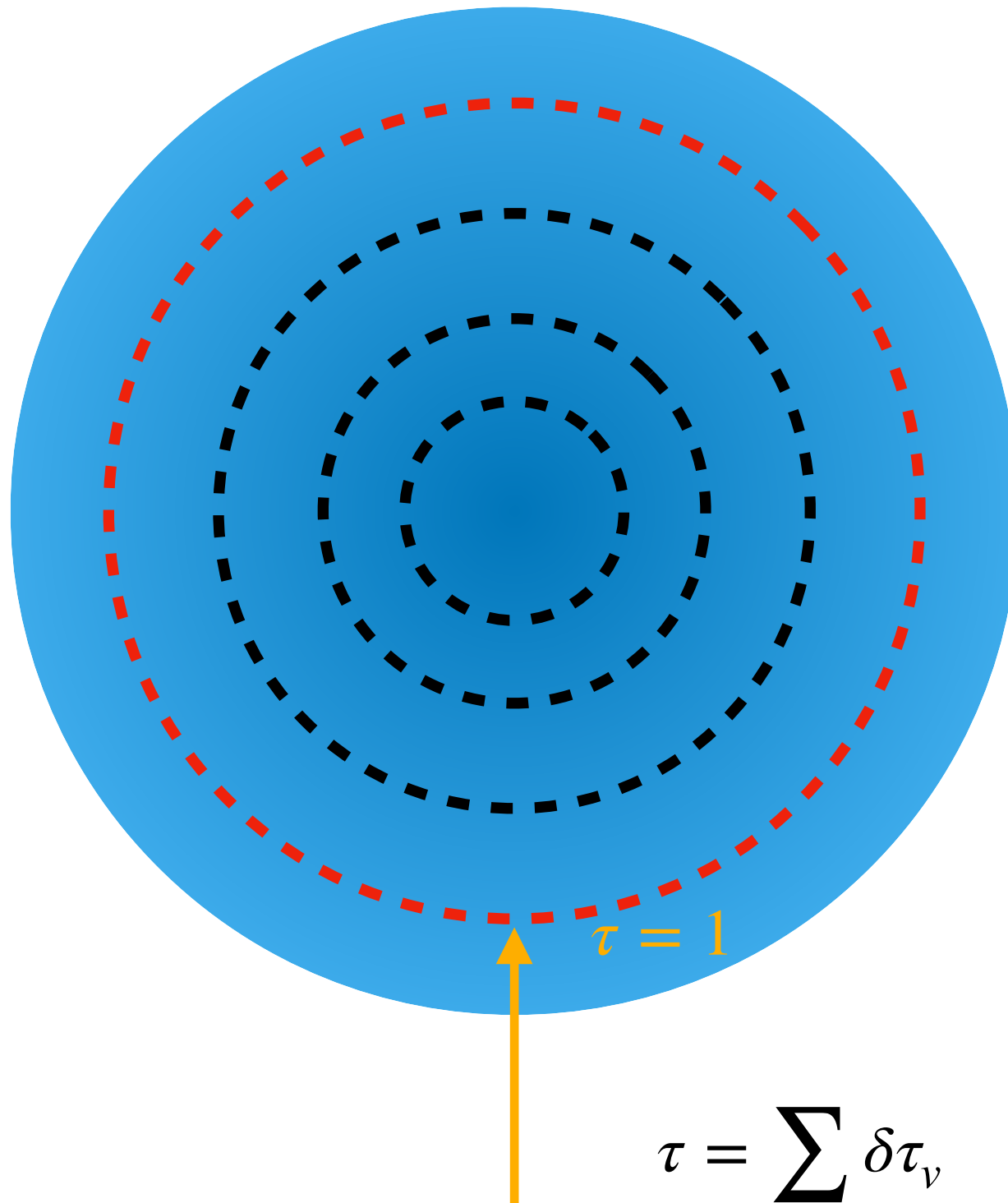
5

Once we have the total luminosity we can find the temperature (Stefan-Boltzmann law)

$$T_{eff} = \left(\frac{L_{tot}}{4\pi\sigma_{SB}R_{phot}^2} \right)^{1/4}$$

But we need the **radius of the photosphere!**

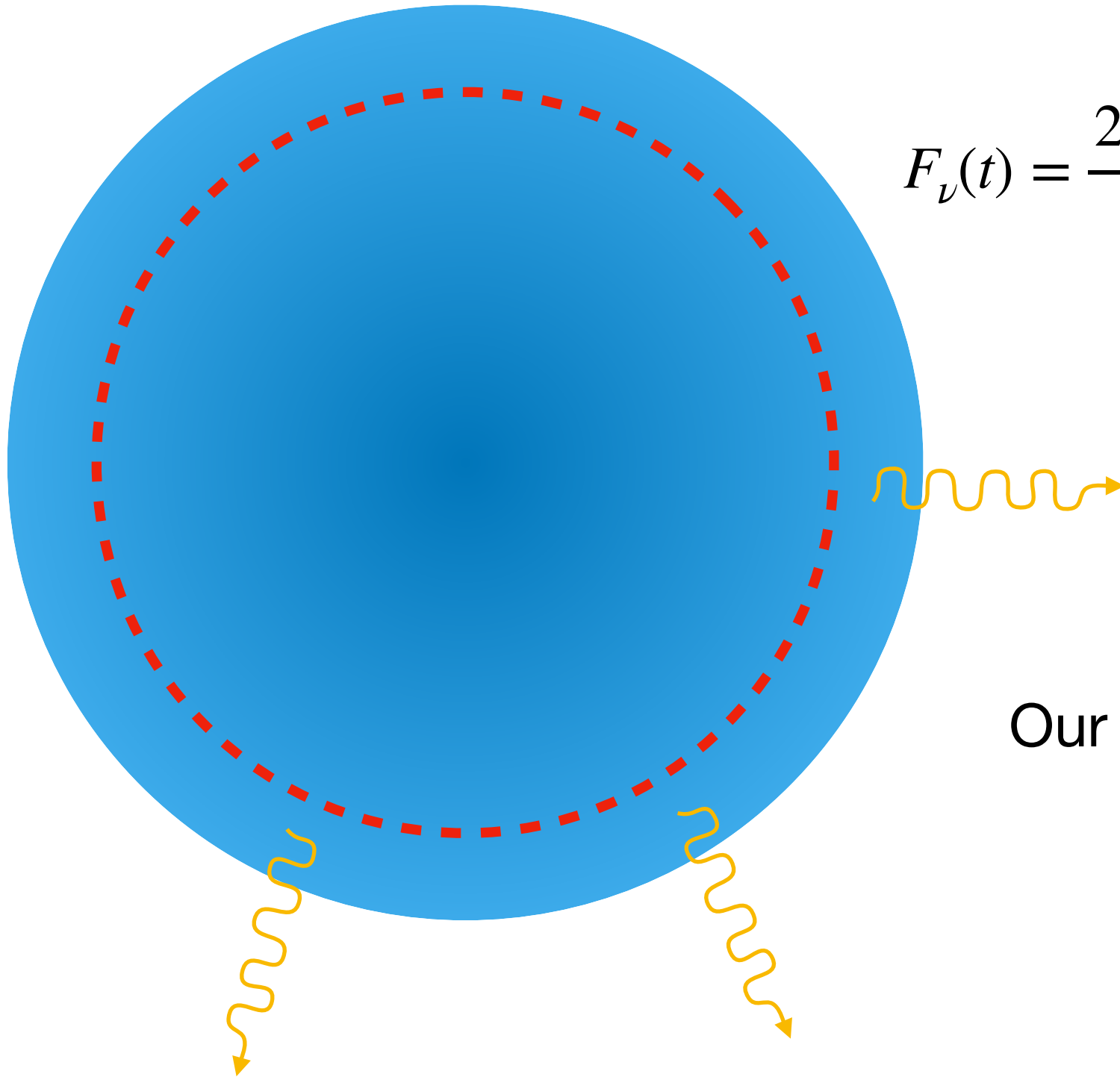
We find it by integrating the optical depth from outside to inside, until it is = 1



6

Now, given a distance, we have a flux for a given frequency!

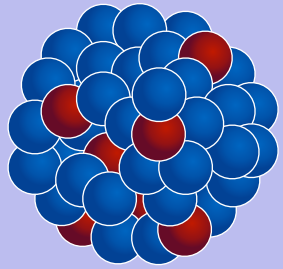
$$F_{\nu}(t) = \frac{2\pi h\nu^3}{c^2} \frac{1}{\exp\left[\frac{h\nu}{k_B T(t)}\right] - 1} \frac{R_{phot}^2(t)}{D^2}$$



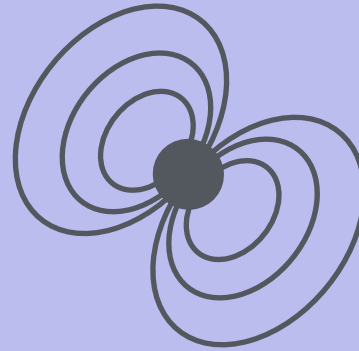
Our Kilonova is ready!

Wait... what about the
heating?

R-process
elements decay

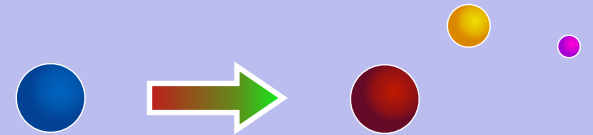


Magnetar
spindown

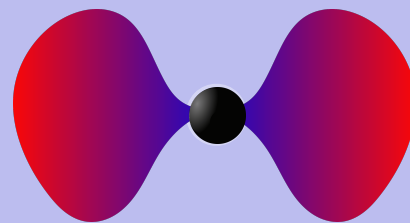


We can use
whatever we can
imagine!

Free neutron decay



Fallback
accretion



*Art by
Vittoria Macioci*

Playing with the Toy Model

- 1) Do we really need heating? What about an initially hot bubble just expanding and radiating?
- 2) Playing with the opacity 1: make a blue and a red kilonova
- 3) Playing with the opacity 2: what about a radial distribution of electron fraction Y_e ?
- 4) Add a thermalization efficiency
- 5) Add a central engine!

Some possible central engine

“Millisecond magnetar” spindown

$$L_{sd} = 7 \times 10^{50} \text{erg s}^{-1} \left(\frac{I}{1.3 \times 10^{45} \text{g cm}^2} \right) \left(\frac{B}{10^{15} \text{G}} \right)^{-2} \left(\frac{P_0}{0.7 \text{ms}} \right)^{-4} \left(1 + \frac{t}{t_{sd}} \right)^{-2}$$

$$t_{sd} \simeq 150 \text{s} \left(\frac{I}{1.3 \times 10^{45} \text{g cm}^2} \right) \left(\frac{B}{10^{15} \text{G}} \right)^{-2} \left(\frac{P_0}{0.7 \text{ms}} \right)^2$$

$$\dot{Q}_{sd} = \epsilon_{th} L_{sd}$$

Fallback accretion

$$\dot{Q}_{fb} = \epsilon_j \dot{M}_{fb} c^2 \simeq 2 \times 10^{51} \text{erg s}^{-1} \left(\frac{\epsilon_j}{0.1} \right) \left(\frac{\dot{M}_{fb}(0.1 \text{s})}{10^{-3} M_{\odot} \text{s}^{-1}} \right) \left(\frac{t}{0.1 \text{s}} \right)^{-5/3}$$

Other decay channels

Nichel decay

$$\dot{\epsilon}_{Ni} = 3.9 \times 10^{10} e^{-t/t_{Ni}} + 6.78 \times 10^9 [e^{-t/t_{Co}} - e^{-t/t_{Ni}}]$$

$$t_{Ni} = 8.8 \text{ days}$$

$$t_{Co} = 113.6 \text{ days}$$